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ANALYSIS OF TANDEM AXLELOADS
BY ELASTIC THEORY

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ABSTRACT

ANALYSIS OF TANDEM AXLELOAD BY ELASTIC THEORY

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AASHO Road Test tandem axleloads were analyzed to determine the magnitude of the tandem axleload that causes the same damage as the 18-kip (80-kN) single axleload. The procedure is outlined and is the same used to analyze single axleloads. The essential findings were as follows:

1. One repetition of a 34-kip (151-kN) tandem axleload appears to cause the same damage as one repetition of an 18-kip (80-kN) single axleload.
2. The relationships of log repetitions versus axleload used by Kentucky appear to be equally valid for single and tandem axleloads.
3. The use of superposition principles, and equivalency of repetitions, in combination with terminal serviceability indices permitted analyses of the tandem axleload data and comparison with equivalent Kentucky thickness designs converted to the AASHTO structural number.
4. Single axleload analyses were combined with the tandem axleload analyses and superimposed by Kentucky equivalent designs. The Kentucky designs, based upon elastic theory, enveloped 91 percent of the AASHO Road Test data.
5. For pavement design, the estimated fatigue for a 20-year design using the AASHTO damage factors will be reached in 16.2 years using the Kentucky damage factors. The Kentucky damage factors more closely approximate observed behavior than the AASHTO factors.

INTRODUCTION

Thicknesses of flexible pavement structures have been determined by several systems, among which are Kentucky's 1948 and 1959 curves (1, 2), the AASHTO Interim Guide (3), and Kentucky's proposed 1973 design guide (4). The objectives of this paper are (1) to determine the magnitude of the tandem axleload that produces the same equivalent damage as one 18-kip (80-kN) single axleload, (2) to develop appropriate damage factors for tandems using the above tandem axleload as the base value, and (3) to compare results obtained from the above design methods.

BACKGROUND

The 1959 Kentucky design curves were based upon empirical tests and observations of pavements in 1948 and 1957 and were generally drawn to separate points representing pavements which were performing satisfactorily from those representing unsatisfactory performance. The 1959 curves were an extension of Kentucky's 1948 curves to account for increased traffic volumes.

The 1973 Kentucky design guide is based upon elastic theory, and the curves (5, 6) were drawn to provide structural thicknesses having the same strain values. The limiting values of strain were obtained by matching theoretical strain values with field performance data. The matching was based primarily upon the performance of a 23-inch (580-mm) pavement composed of 7 3/4 inches (195 mm) of asphaltic concrete and 15 1/4 inches (395 mm) of dense-graded aggregate placed on a soil having a CBR of 7 to withstand a field loading of 8×10^6 applications of 18-kip (80-kN) axleloads.

A stated objective of the AASHO Road Test (7) was to determine significant relationships between the numbers of repetitions of specified axleloads of different magnitudes and arrangements and the performance of different asphaltic concrete pavement structures. The emphasis was upon pavement structural fatigue analysis and comparison of load-damage effects.

To make a direct comparison of design thicknesses obtained by the AASHTO system (3) and the Kentucky system (4) required expressing the two systems in equivalent terms. Fatigue data for each pavement section of the Road Test formed the basis for the AASHTO thickness design nomographs. The following equation was used to express the relationship between the number of repetitions (N), structural number (SN), and level of serviceability (P_t):

$$\log N = 5.93 + 9.36 \log (SN + 1) - 4.79 \log (L_x + L_2) + 4.33 \log L_2 + G_t/B_x, \quad 1$$

where $G_t = \log[(4.2 - P_t)/(4.2 - 1.5)]$,

$$B_x = 0.40 + 0.08 (L_x + L_2)^{3.23}/(SN + 1)^{5.19} L_2^{3.23},$$

$L_x =$ any axleload (in kips), and

$L_2 =$ 1 for single axleloads and

$=$ 2 for tandem axleloads.

Comparison of seasonally weighted data (7) with Equation 1 was deemed appropriate and necessary. The following methodology was used in Kentucky to analyze single axleload data from the AASHO Road Test (7, 8):

1. Layer thicknesses for each test section were converted to a corresponding structural number by

$$SN = a_1 d_1 + a_2 d_2 + a_3 d_3, \quad 2$$

where d_1 , d_2 , and d_3 are thicknesses of the surface, base, and subbase layers in the pavement structure and a_1 , a_2 , and a_3 were assumed to be 0.44, 0.14, and 0.11, respectively.

2. Graphs were made of level of serviceability versus number of axleload repetitions for each section of each loop.

3. Graphs for various P_t 's were made for SN versus N (Figure 1).

4. The 1973 Kentucky damage factors (DF) were calculated using

$$DF = (1.2504)^{(P - 18)}, \quad 3$$

where P is the single axleload in kips and the constant 1.2504 is the slope of the straight-line, single axleload-log repetitions relationship; the base or reference axleload was taken to be 18 kips (80 kN).

5. For a given P_t , the products of N and DF were plotted on the graph for 18-kip (80-kN) data from Lane 1 of Loop 4. This procedure converts (by superposition) the number of repetitions of a given axleload to an equivalent number of repetitions of an 18-kip (80-kN) axleload. The corresponding SN is not changed.

6. For single axleloads, Equation 1 reduces to

$$\log W_t = 5.93 + 9.36 \log (SN + 1) - 4.79 \log (L_x + 1) + G_t/B_x. \quad 4$$

Solutions of Equation 4 were superimposed on the graphs of Step 3.

7. Studies had determined that the soil used at the AASHO Road Test had a CBR of 5.2 as determined by the Kentucky CBR test method. Thicknesses (4) corresponding to a CBR of 5.2 and an asphaltic concrete modulus of 480 ksi (3.31 GPa) were converted to SN's by Equation 2 and superimposed on graphs from Step 6. For P_t of 2.0 and 2.5, it was noted that Equation 4 approximated a best fit of the data, and the Kentucky design method required thicknesses (in terms of SN) greater than that required by approximately 93 percent of the Road Test single axleload fatigue data. For other P_t graphs, the Equation 4 trend line and the Kentucky design curve reasonably fitted some portions, but not all, of any range of data -- except for the 18-kip (80-kN) data at a P_t of 2.5.

8. The relationship for single axleloads shown in Figure 2 was used to choose the appropriate P_t - axleload combination to be superimposed on the 18-kip (80-kN) base graph for P_t of 2.5 (Figure 3).

9. The following observations were made:

a. The 480-ksi (3.31-GPa) asphalt concrete modulus Kentucky curve in Figure 3 requires an SN greater than that required by approximately 93 percent of all AASHO single axleload data points (Kentucky moduli lines for 270, 305, 375, and 600 ksi (1.86, 2.10, 2.59, and 4.14 GPa) have also been superimposed).

b. The 18-kip (80-kN) single-axle curve for $P_t = 2.5$ from Equation 4 was superimposed onto Figure 3, showing that the SN for 59 percent of the data points was less than that from Equation 4, has the same general shape suggested by the data points except for the 6-kip (27-kN) set, and is almost a direct fit of the family of asphaltic concrete elastic moduli lines.

c. The combination of a and b above confirms that the 1973 Kentucky design guide (4) does provide for variable terminal serviceability levels, as a function of the EAL level, with a high probability of success in the design life.

d. Elastic theory provides a sound basis for accurately describing the observed performance data of both Kentucky highways and the AASHO Road Test.

e. One level of terminal serviceability should not be used for the wide range of EAL's, as suggested by AASHTO thickness design nomographs.

f. AASHTO nomographs provide for approximately a 60-percent probability of successful pavement performance throughout the design life.

g. The semi-logarithmic relationship between single axleload and repetitions used in the 1973 Kentucky design guide is not only a reasonable assumption but has been confirmed by the analysis of the AASHO Road Test data (7).

Kentucky had not developed its own damage factors for tandem axleloads, nor had it determined the magnitude that produces the same damage as one repetition of an 18-kip (80-kN) single axleload. The purpose of this investigation is to provide these answers.

ANALYSIS

Single-Tandem Axleload Equivalency

Analysis of the single axleload data (8) from the AASHO Road Test (7) proved to be rewarding enough to warrant an analysis of the tandem axleload data (*Appendix A, 7*). AASHTO (3) had determined that an 18-kip (80-kN) single axleload and a 33.124-kip (147-kN) mean tandem axleload both produced the same damage. However, it was noted that the "equivalent" tandem axleload varied and was a function of SN and P_t . Again, it was deemed appropriate and desirable to investigate the tandem axleload data

by the same Steps 1 through 3 used to analyze the single axleloads. Thus, the 32-kip (142-kN) tandem axleload data were plotted for P_t of 1.5, 2.0, 2.5, 3.0, and 3.5; Figure 4 for $P_t = 2.5$ is an example. The dashed line is the solution of Equation 1, and the 480-ksi (3.31-GPa) line is the same Kentucky solution shown in Figure 1. The similarity between Figures 1 and 4 prompted plotting of the 24-kip (107-kN), 40-kip (178-kN), and 48-kip (214-kN) tandem axleload data for the five values of P_t mentioned above. The same trends mentioned earlier were evident; this is represented by the latter portion of Step 8, except the reference axleload was 32 kips (142 kN).

To proceed to Step 4, the first task was to determine what tandem axleload produced the same equivalent damage as an 18-kip (80-kN) single axleload. The criteria to be satisfied were

1. the level of serviceability must be equal,
2. the same number of repetitions must be used,
3. one value of an AASHTO SN must be chosen to evaluate the equivalent effects of single and tandem axleloads, and
4. the tandem axleload damage factors must be compatible with the 1973 Kentucky (4) single axleload damage factors.

Kentucky has associated 18-kip (80-kN) single axleloads and a level of serviceability of 2.5 together. Having determined (8) that the 1973 Kentucky design guide (4) and the AASHO Road Test results (7) were compatible, the same serviceability value of 2.5 was chosen. As shown in Figures 1 and 4, the AASHO Road Test data (7) at a serviceability of 2.5 had a span of repetitions from 1,000 to 1,000,000; and 100,000 repetitions was chosen for analysis only because that point is the middle of the log scale. From Figure 1, the 480-ksi (3.31-GPa) line intersects the 100,000 repetitions line at a SN value of 3.07. Thus, the criteria values were determined.

The 480-ksi (3.31-GPa) line exceeded the SN for 93 percent of the AASHO Road Test single axleload data (Figure 3); and, as shown in Figure 1, one data point was above the 480-ksi (3.31-GPa) line. The 480-ksi (3.31-GPa) line was superimposed on Figure 4, and similar figures for a serviceability value of 2.5 and tandem axleloads of 24 kips (107 kN), 40 kips (178 kN), and 48 kips (214 kN) were prepared. A line of equal offset to the 480-ksi (3.31-GPa) line was constructed such that one data point was above the constructed line. The SN value at 100,000 repetitions was graphically determined and transferred to Figure 5. These four pairs of axleload-SN values were connected with a smooth curve. Thus at a SN = 3.07, Figure 5 shows that an 18-kip (80-kN) single axleload would be equivalent to a 34-kip (151-kN) tandem axleload.

Tandem Axleload Equivalency Factors

Tandem axleload equivalency factors must be compatible in methodology with the single axleload

damage factors. Thus, the Kentucky single axleload relationship of axleload versus log repetitions (Figure 2) required the use of the same axleload versus log repetitions format for tandem axleloads. Also, Kentucky has associated 8,000,000 repetitions of 18-kip (80-kN) single axleloads as a reference pavement. Thus, a 34-kip (151-kN) tandem axleload must be associated with 8,000,000 repetitions on the same pavement.

Equation 5 was used to express the single axleload-versus-repetitions relationship labeled as the 1973 Kentucky line in Figure 6:

$$EAL = N(1.2504)^{(P - 18)}. \quad 5$$

Setting EAL equal to 8,000,000 and P (single axleload) equal to zero, N was determined to be 446,654,133 repetitions. Assuming this same intercept value for a tandem axleload and using the point for a tandem axleload of 34 kips (151 kN) at 8,000,000 repetitions, the resulting straight line in Figure 6 is given by

$$EAL = N(1.1254)^{(P - 34)}. \quad 6$$

Here, the constant 1.1254 is the slope of the tandem axleload-log repetitions relationship. The tandem axleload equivalency factors shown in Figure 7 were derived as Step 4 from

$$\text{Traffic Equivalency Factor} = N_{34}/N_p, \quad 7$$

where N_{34} = number of repetitions for 34-kip (151-kN) axleload and

N_p = number of repetitions for any other given tandem axleload.

Equation 7 can be expressed in the form of Equation 3 as

$$DF = (1.1254)^{(P - 34)}. \quad 8$$

The traffic equivalency factor relationships for the 1973 Kentucky tandem axleload and the 1972 AASHO Interim Guide tandem axleload at a SN = 5.0 and level of serviceability of 2.5 are included in Figure 7.

AASHO Serviceability Ratings

Figure 3 is the result of combining data for five pairs of axleload and serviceability levels (Step 5) and is shown in Figure 2 as the line labeled as single axleload. The Kentucky 480-ksi (3.31-GPa) line (Step 7) and the line from Equation 4 (Step 6) are superimposed.

Tandem axleload data were processed in the same manner as the single axleloads except that the reference axleload was 34 kips (151 kN). In Step 6, data for Lane 2 of Loop 4 were used as the base graph. In Step 8, the tandem relationship shown in Figure 2 was used.

The tandem-axleload serviceability-level relationship shown in Figure 2 was used to determine the serviceability level for the four levels of tandem axleloads used at the AASHO Road Test (Step 8). Road Test data (7) were plotted for each axleload (Step 3), and the repetitions at the respective serviceability levels were interpolated and plotted in a manner similar to Figure 4. Using the 1975 Kentucky

traffic equivalency factors for tandem axleloads shown in Figure 7, the interpolated data were equated to equivalent 34-kip (151-kN) repetitions as shown in Figure 8. Approximately 89 percent of the AASHO Road Test tandem data required a SN value less than the Kentucky 480-ksi (3.31-GPa) line while only 47 percent was beneath the line obtained for tandem axleload solutions of Equation 1.

Figure 9 is a combination of single and tandem axleload data from Figures 3 and 8. The Kentucky 480-ksi (3.31-GPa) line is an envelope for 91 percent of the combined AASHO Road Test data while the Equation 1 solutions require a SN in excess of that required by approximately 50 percent of the data.

Net Effect of Kentucky Traffic Equivalency Factors

To determine the net effect of the new, Kentucky single and tandem axleload traffic equivalency factors, data in Kentucky's W-4 tables for 1959 through 1973 were analyzed. The number of axleloads of various groupings of loaded and empty trucks and truck combinations of each type weighed were summed by axleload groups within the single and tandem axleload divisions. These sums were multiplied by the corresponding traffic equivalency factors for the 1959 Kentucky, 1973 Kentucky design systems, and the 1974 AASHTO Interim Guide factors for SN of 5.0 and serviceability level of 2.5. The total accumulated EAL's are shown in Figure 10, indicating that the AASHTO (3) factors would produce a 20-year design level which would be reached within 17.5 years using the 1959 Kentucky factors and 16.2 years using the 1973 Kentucky factors.

Until this current analysis, there has not been a set of tandem axleload equivalency factors for application in Kentucky. The practice has been to weigh the individual axles of the tandem arrangement and use the single axleload equivalency factor for each axleload. Thus, the calculated total accumulated EAL's have been higher than the 1974 AASHTO (3) but less than the 1973 Kentucky values.

SUMMARY

The AASHO Road Test tandem axleload data were analyzed using the same procedure reported previously (8) for a single axleload analysis. The following are the major points in this report:

1. All other factors being equal, one tandem axleload of 34 kips (151 kN) was determined to produce the same fatigue damage as one 18-kip (80-kN) single, rear axleload.
2. The logarithm of repetitions-versus-axleload relationship used by Kentucky (5, 6) appears to be equally as valid for tandem axleloads as for single axleloads.
3. A correlation of serviceability index with axleload was developed for single and tandem axleloads.
4. The combination of a variable terminal serviceability with axleloads and the equation of single

and tandem axleloads permitted combining the AASHO Road Test single and tandem axleload-versus-repetitions data (Figure 9).

5. The superpositioned Kentucky design curve requires SN values in excess of the SN for 91 percent of the combined single and tandem axleload data from the AASHO Road Test.

6. Pavements designed by the 1974 AASHTO thickness design nomographs have been shown to reach the design fatigue level in approximately 80 percent of the expected life when adjusted for increased volume. This discrepancy is explainable by the 1973 Kentucky axleload damage factors.

RECOMMENDATIONS

The AASHTO damage factors used in the Kentucky W-4 tables should be replaced with the following factors:

for Single Axleloads:

$$\text{Damage Factor} = (1.2504)^{(P - 18)}$$

for Tandem Axleloads:

$$\text{Damage Factor} = (1.1254)^{(P - 34)}.$$

ADDITIONAL RESEARCH IN PROGRESS

Current efforts beyond the scope of this report utilizes superposition principles, elastic theory, and strain energy equations to analyze the AASHO Road Test loads and configurations of wheel loads applied to various pavement thicknesses in order to separate the fatigue effects due to front axles from that caused by rear axles. Increased use of wide tires on the front axle and trailers having three or four axles in one group requires additional analyses to determine the appropriate equivalent loads and associated damage factors.

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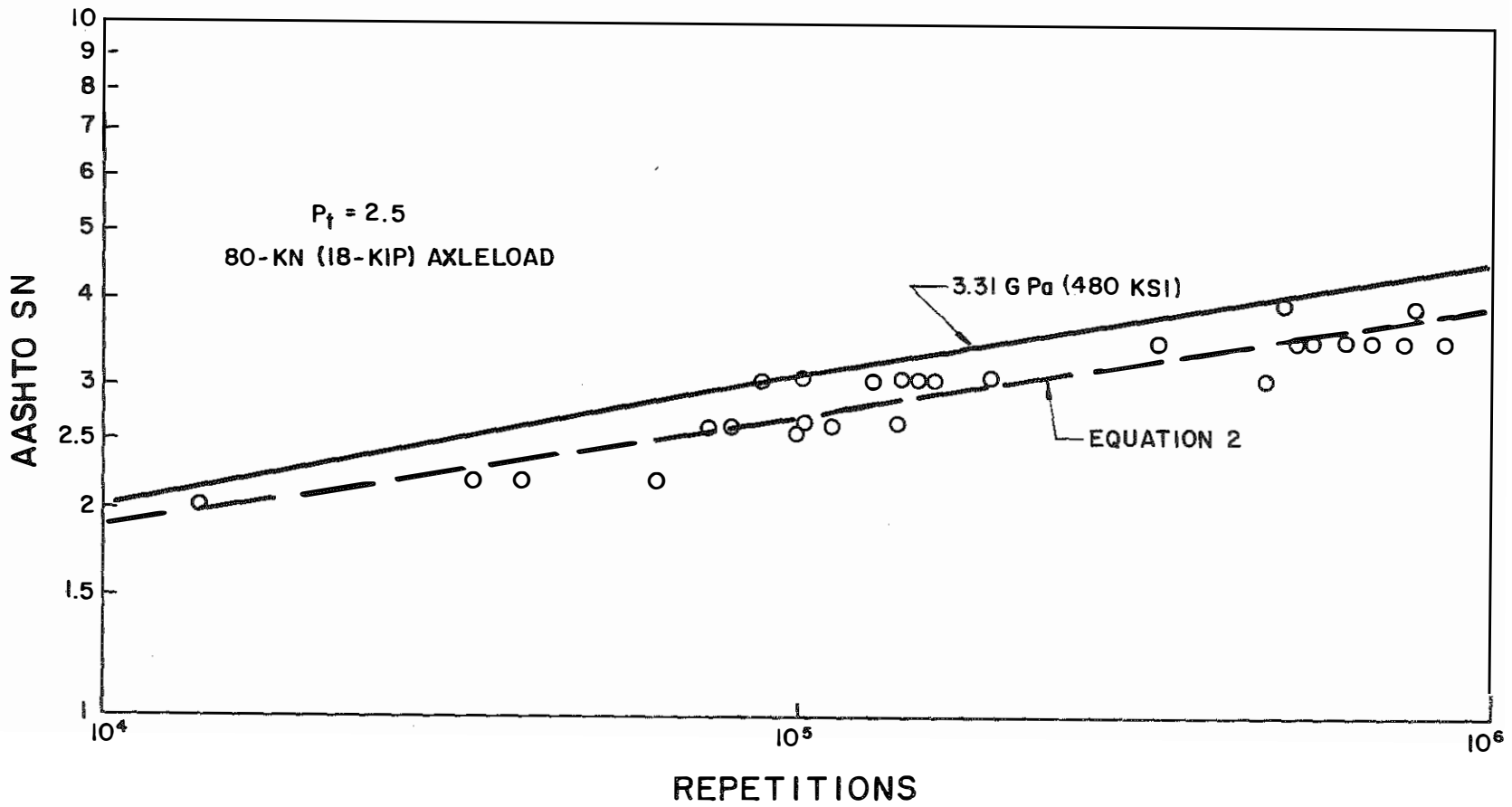


Figure 1. Relationship between Repetitions of an 18-kip (80-kN) Single Axleload and Structural Number for Terminal Serviceability of 2.5, AASHTO Road Test Seasonally Weighted Data (Reference 7, Appendix A).

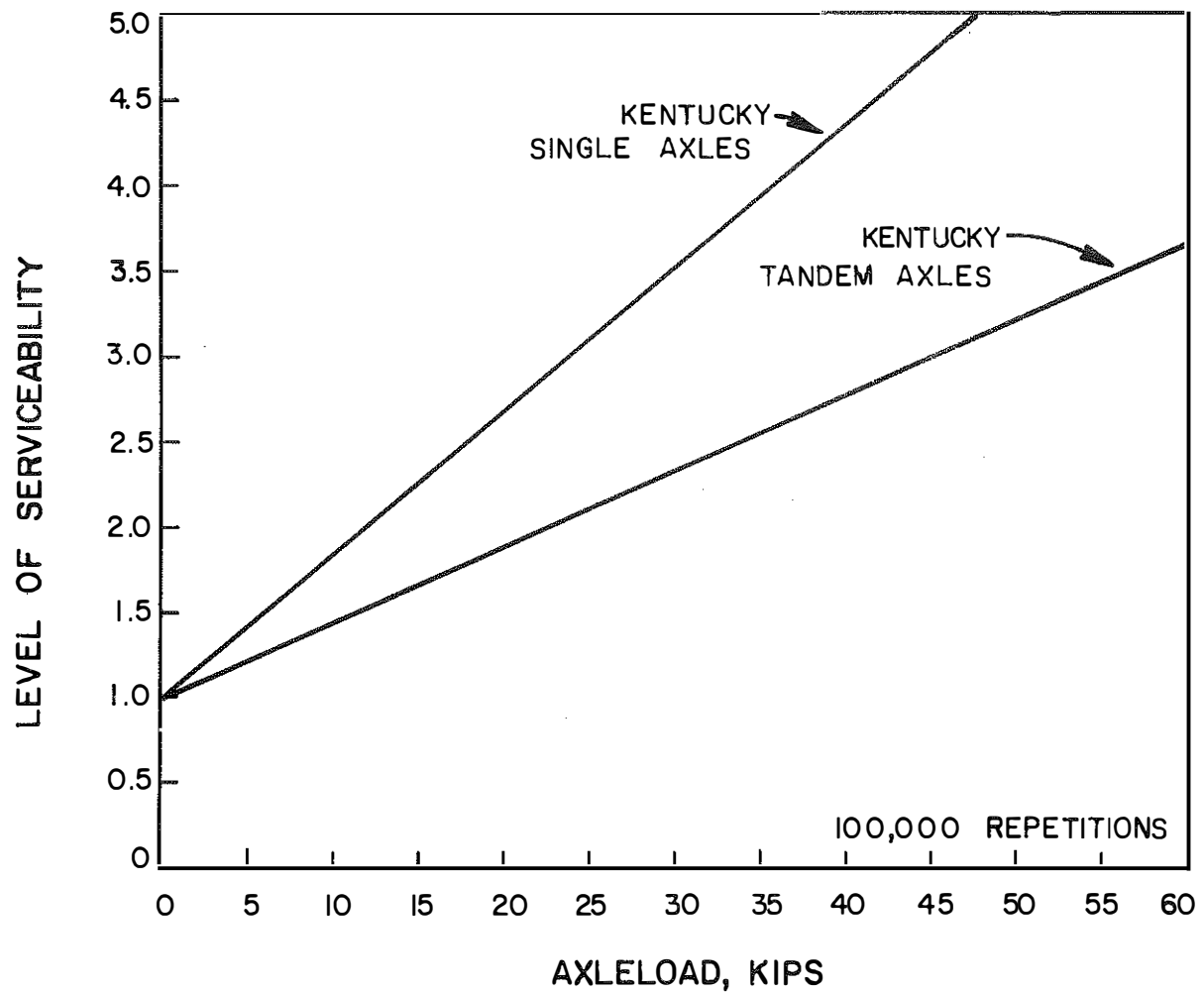


Figure 2. Level of Serviceability versus Axleload.

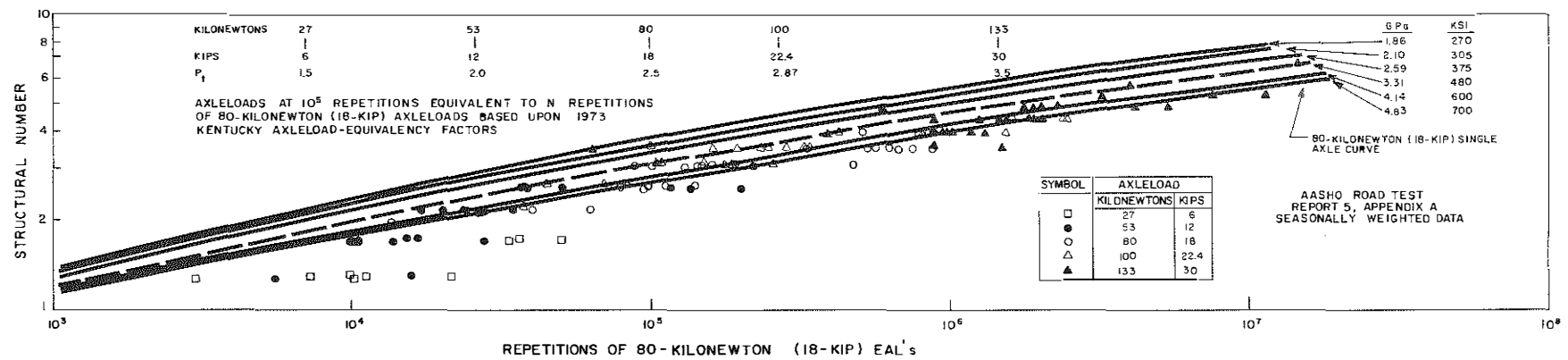


Figure 3. Structural Number versus Repetitions of 18-kip (80-kN) EAL's.

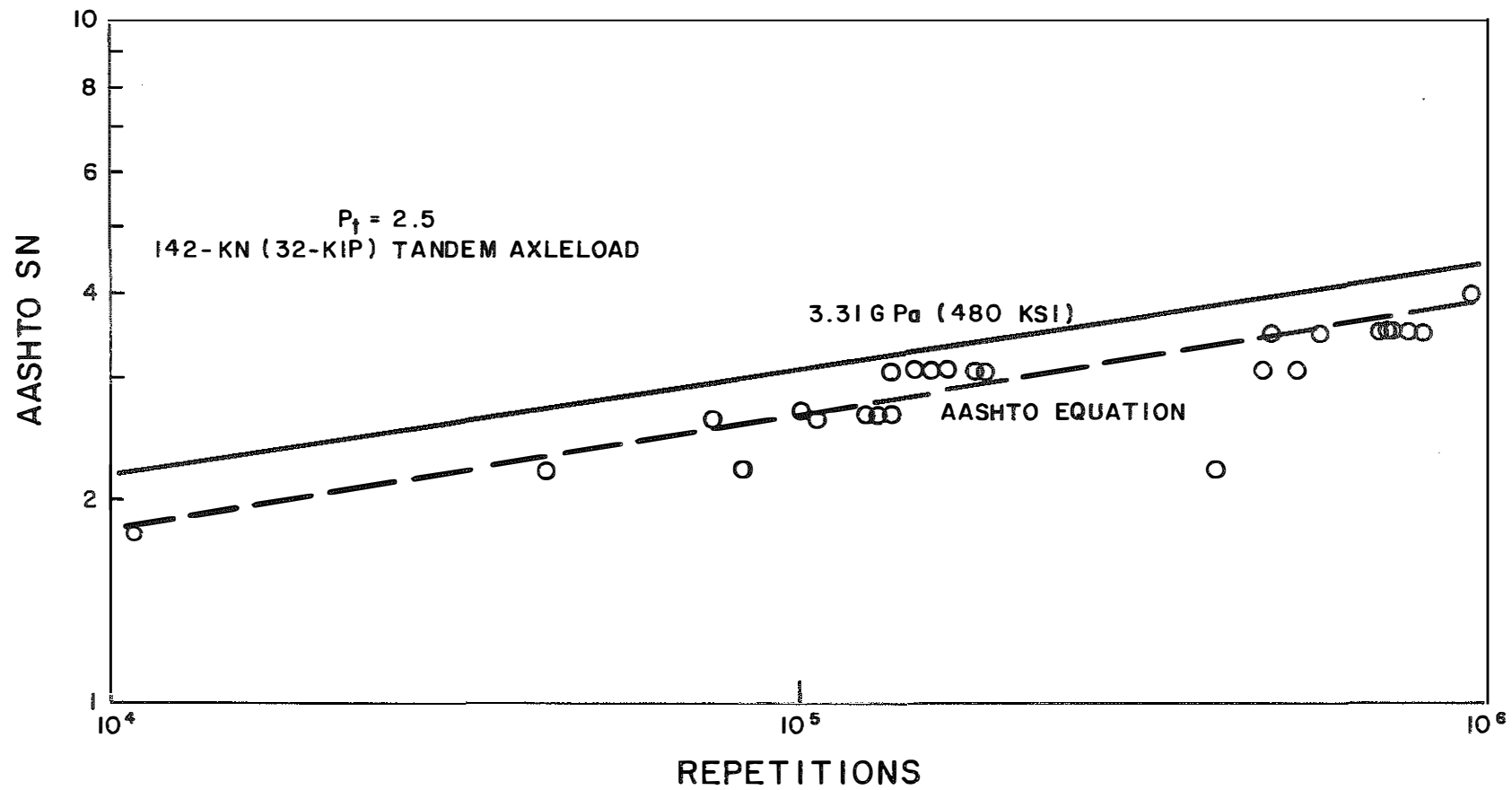


Figure 4. Relationship between Repetitions of a 34-kip (151-kN) Tandem Axleload and Structural Number for Terminal Serviceability of 2.5; AASHTO Road Test, Seasonally Weighted Data (Reference 7, Appendix A).

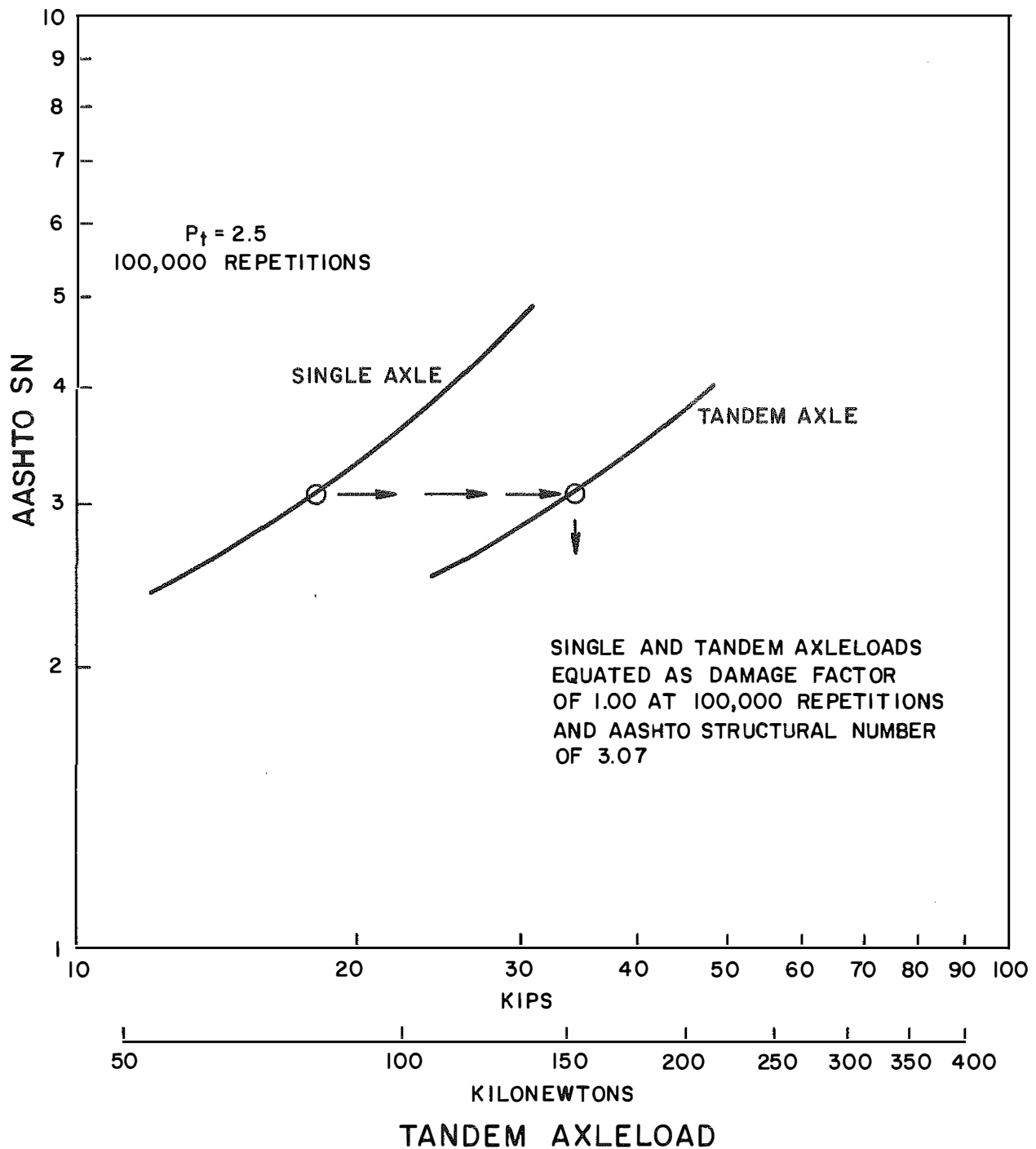


Figure 5. Relationship between AASHTO Structural Number and Axleload.

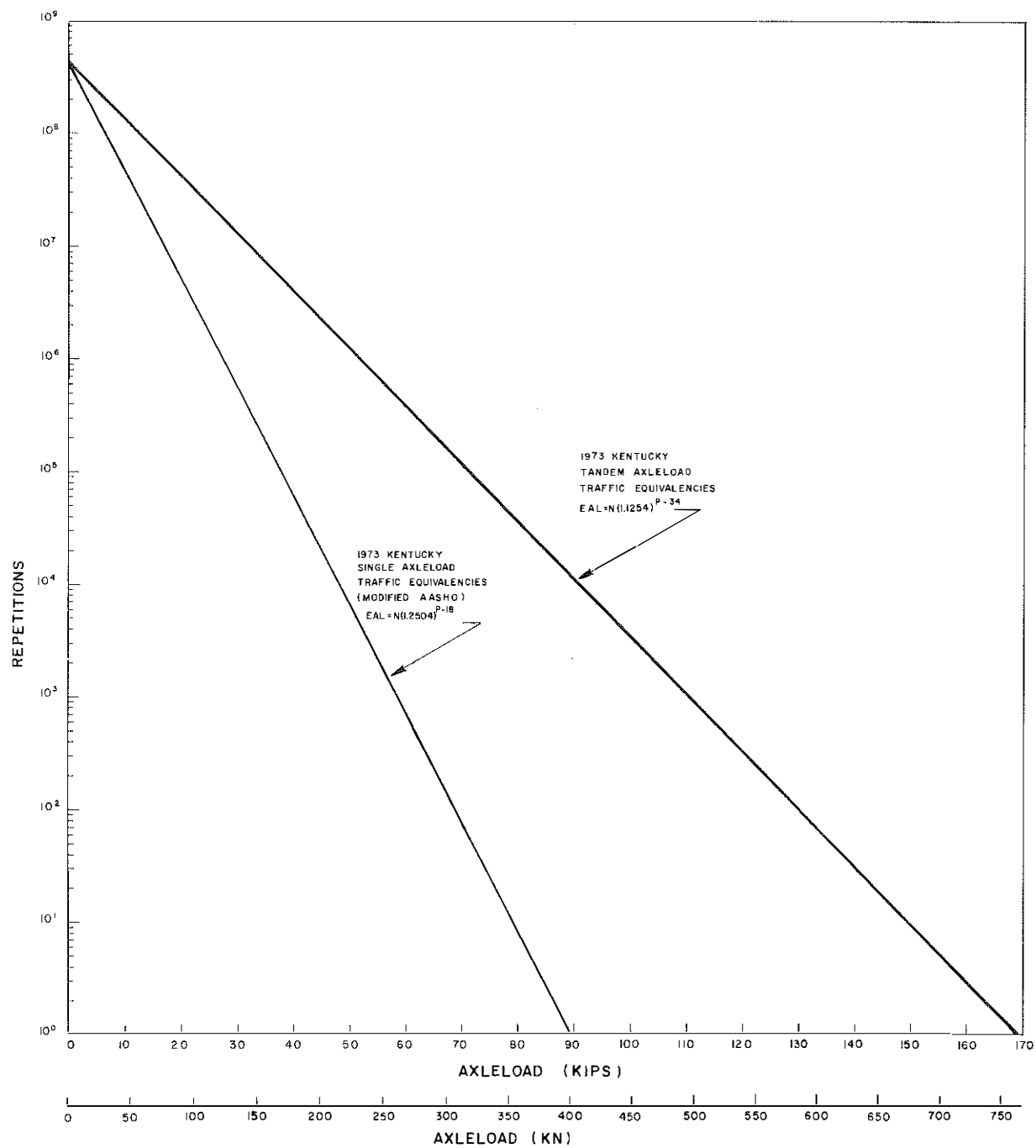


Figure 6. Repetitions versus Axleload Equivalencies.

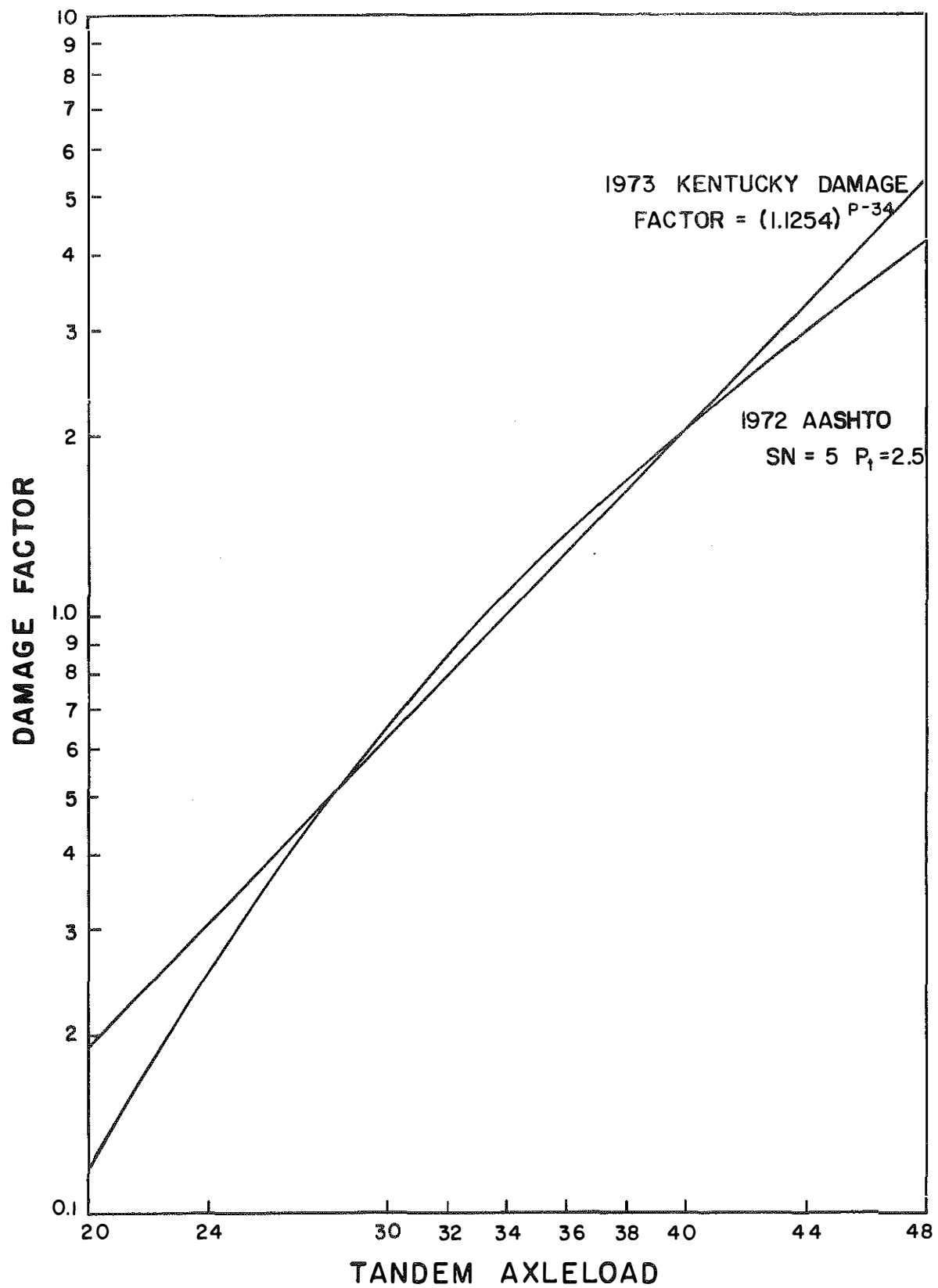


Figure 7. Damage Factor versus Tandem Axleload.

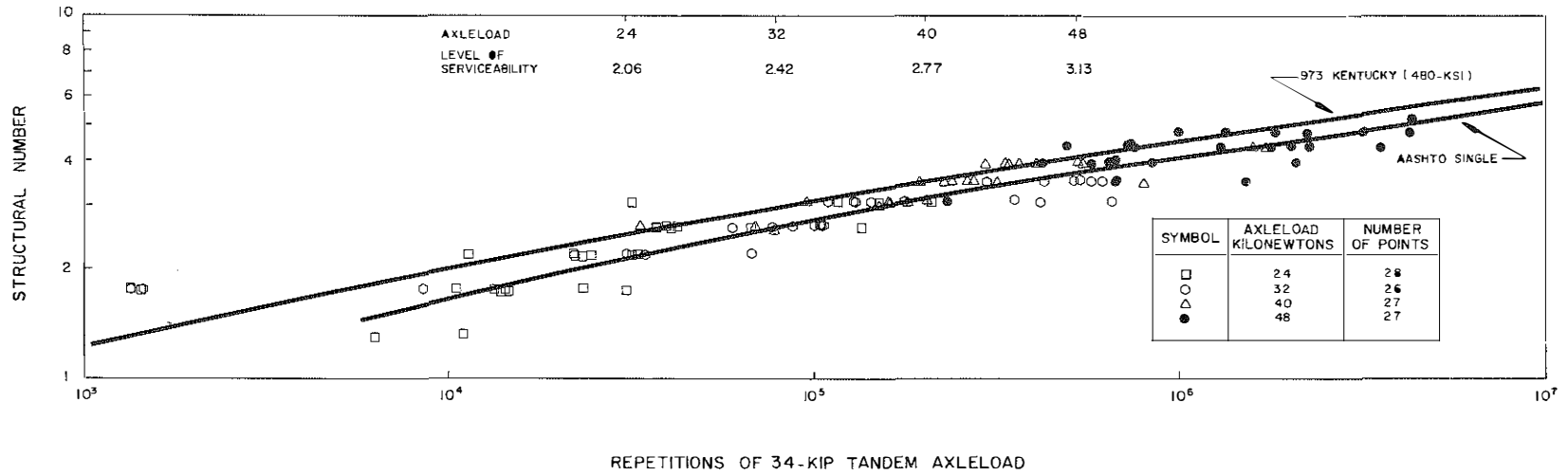


Figure 8. Structural Number versus Repetitions of 34-kip (151-kN) Tandem Axleload.

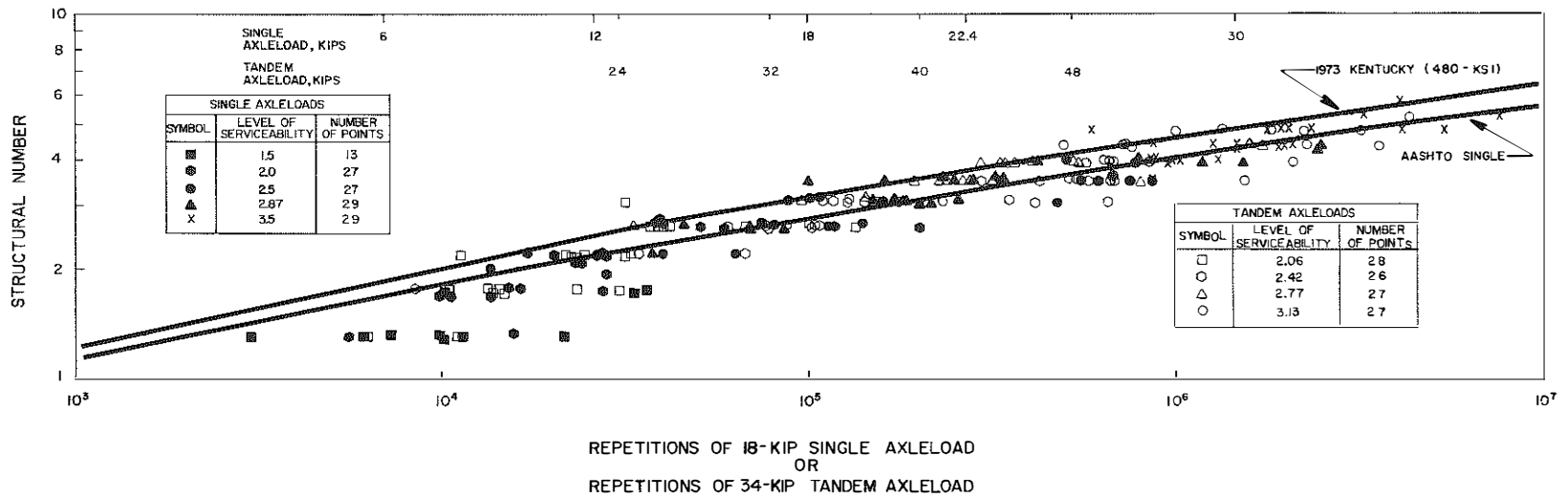


Figure 9. Structural Number versus Repetitions of 18-kip (80-kN) Single Axleload or Repetitions of 34-kip (80-kN) Tandem Axleload.

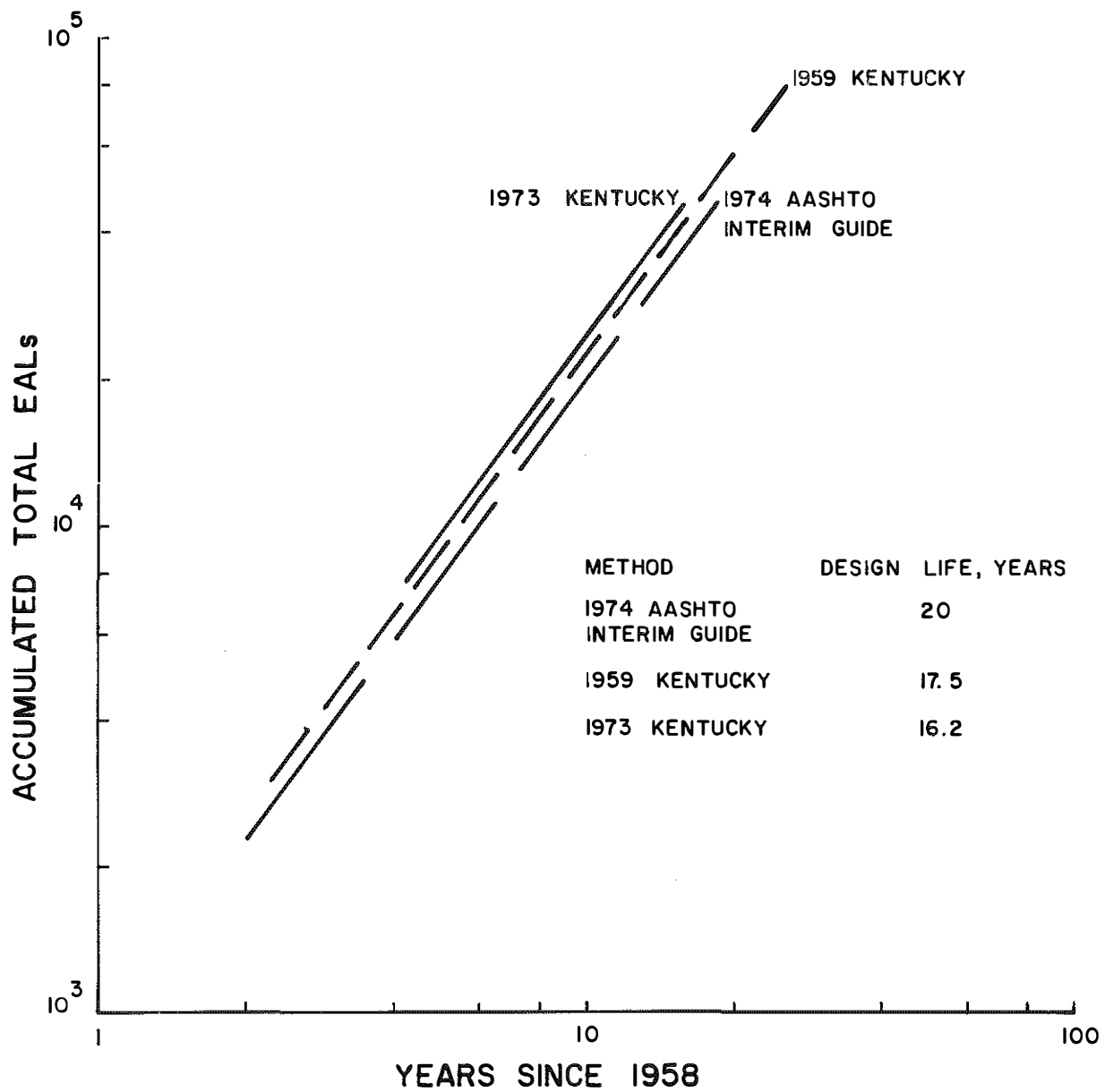


Figure 10. Accumulated, Total EAL'S versus Years since 1958.